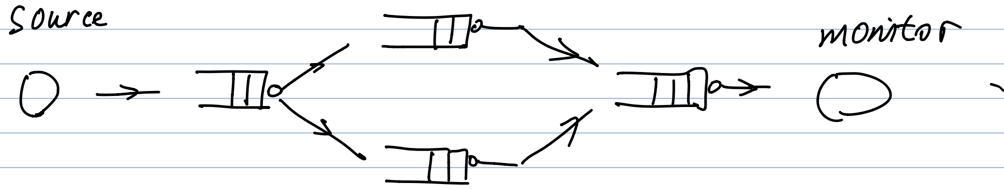
Lecture 17

Age-optimal Scheduling in Queues (4)

multi-hop queueing network:



- i.i.d. exponential service times, preemptive LGFS is age optimal.

  Usually packet arrivals are out of order.

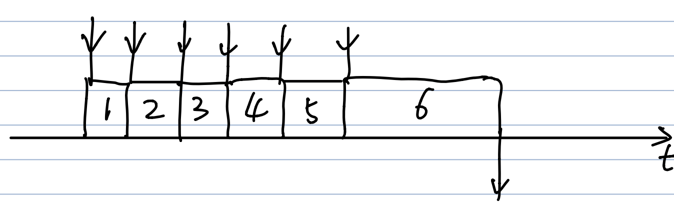
- i.i.d. NBU service times, non-preemptive LGFS is near age optimal.

Why non-preemptive policies are preferred for NBU distributions?

Recall: $R = [X - t \mid X > t]$.

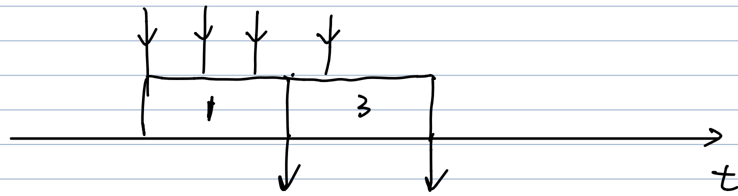
$R \leq_{st} X$

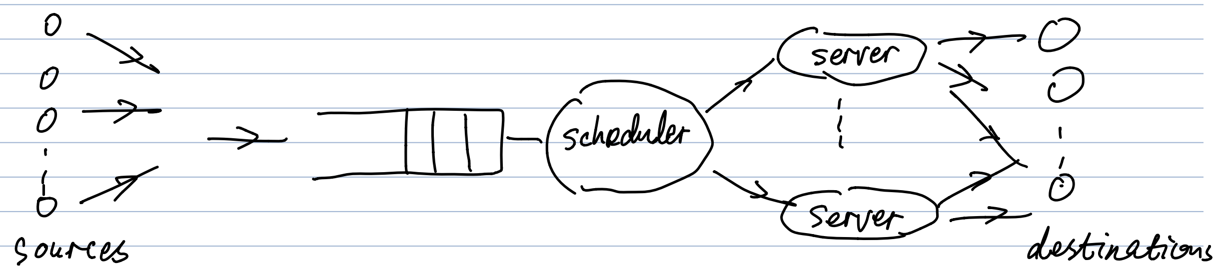
preemptive LGFS:



no delivery for a long time period, if λ is high.

non-preemptive LGFS:





Packet i of flow n. generation time. $S_{n,i}$ arrival time, $C_{n,i}$ delivery time $D_{n,i}$

$$0 \le S_{n,1} \le S_{n,2} \le \cdots$$

$$S_{n,i} \le C_{n,i} \le D_{n,i}$$

Def: Synchronized arrivals:

There exist S_i and C_i, such that

$$S_{n,i} = S_i, \quad C_{n,i} = C_i \quad \forall n.$$

B: buffer size.

M: No. of servers.

If $B=0$, system can keep M packets,

i.i.d. expenential service times,

AoI:

$$\Delta_n(t) = t - \max\{S_{n,i} : D_{n,i} \le t\}.$$

$$\vec{\Delta}(t) = (\Delta_1(t), \cdots, \Delta_N(t)).$$

symmetric age metrics:

functions $P(\cdot)$:

$$P \circ \vec{\Delta} = P(\vec{\Delta}) = P(\Delta_{[1]}, \Delta_{[2]}, \cdots, \Delta_{[N]}).$$

e.g.

$$P_1(\vec{\Delta}) = \frac{1}{N}\sum_{n=1}^{N} \Delta_n.$$

$$P_2(\vec{\Delta}) = \max_{n=1,\cdots,N} \Delta_n.$$

$$P_3(\vec{\Delta}) = \sum_{n=1}^{N} g(\Delta_n).$$

process of symmetric age penalty function:

$$\{P \circ \vec{\Delta}_\pi(t), t \geq 0\}.$$

Good scheduling policy:

Def: Maximum Age First (MAF): the flow with maximum age is served first, with ties broken arbitrarily.

Def: Maximum Age First, Last Generated First Served (MAF-LGFS).

the last generated packet from the flow with the maximum age is served first.

Thm:

If (i) there is a single server ($M=1$),

(ii) synchronized arrivals,

(iii) i.i.d. exponential service times,

then for all $B \geq 0$, $\mathcal{I}$, symmetric increasing function p, and $\pi \in \Pi$,

$$[\{p \circ \vec{\Delta}_{prmp\text{-}MAF\text{-}LGFS}(t), t \geq 0\} | \mathcal{I}]$$

$$\leq_{st} [\{p \circ \vec{\Delta}_{\pi}(t), t \geq 0\} | \mathcal{I}],$$

Reading: Section 2.4 of the book.

Sun, Uysal, Kompella, AoI Workshop 2018.

Def: Maximum Age of Served Information First, Last Generated, First Served (MASIF-LGFS):

Last generated packet from the flow with the maximum age of served information is served first, with ties broken arbitrarily.

Thm:

If (i) synchronized arrivals.

(ii) i.i.d. NBU service times.

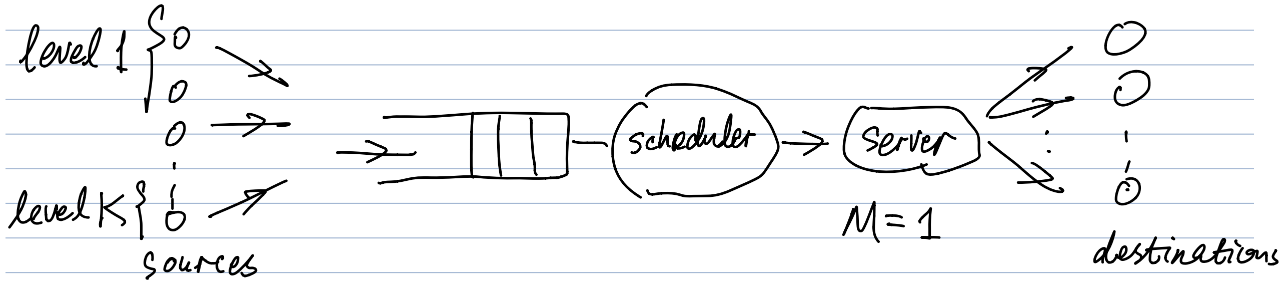
then for all M, $B \geq 0$, $\mathcal{I}$, symmetric increasing function p, and $\pi \in \Pi$,

$$\left[\{p \circ \vec{W}_{\text{non-prmp, MASIF-LGFS}}(t), t \geq 0\} \mid \mathcal{I}\right]$$

$$\leq_{st} \left[\{p \circ \vec{\Delta}_{\pi}(t), t \geq 0\} \mid \mathcal{I}\right].$$

Reading: Section 2.4 of the book.

Sun, Uysal, Kompella, AoI Workshop 2018.



Multiple priorty levels:

Def: Informative packet:

Age of an informative packet is smaller than age of information.

Def: Preemptive Priority (PP):

Among flows with informative packets. the flows with the highest priority are served first.

Def: PP-MAF-LGFS:

priorty level

flows with the same priority level

packets in a flow

Def: lexicographic optimality (lex-optimal),

① find a set of optimal policies Π_{opt} for high priority flows.

② within Π_{opt}, find a set of optimal policies for low priority flows.

Thm:

If (i) there is a single server ($M = 1$),

(ii) synchronized arrivals for flows in each priority level.

(iii) i.i.d. exponential service times.

then. the policies PP-MAF-LGFS is lex-age-optimal.

Corollary.

If each priority level has a single flow, then the arrival process is arbitrary.

Reading: Maatouk, Sun, Ephremides,
Assaad, WiOpt 2020.

Course Presentation. Project.